

The Pure Expectations Theory and Estimation of Forward Rates

In Chapter 4, we saw that the shape of the yield curve depends primarily on two factors: (1) expectations about future inflation and (2) the relative risk of securities with different maturities. We also saw how to calculate the yield curve, given inflation and maturity-related risks. In practice, this process often works in reverse: Investors and analysts plot the yield curve and then use information embedded in it to estimate the market's expectations regarding future inflation and risk.

This process of using the yield curve to estimate future expected interest rates is straightforward, provided: (1) we focus on Treasury bonds, and (2) we assume that all Treasury bonds have the same risk (in other words, there is no maturity risk premium). Although this second assumption may not be reasonable, it enables us for the time being to take out the effects of risk and focus exclusively on how expectations about future interest rates affect the shape of the yield curve. Later on, we will show what happens when we once again assume that there is a maturity risk premium.

In fact, even though most evidence suggests that there is a positive maturity risk premium, some academics and practitioners contend that this second assumption is reasonable, at least as an approximation. They argue that the market is dominated by large bond traders who buy and sell securities of different maturities each day, that these traders focus only on short-term returns, and that they are not concerned with maturity risk. According to this view, a bond trader is just as willing to buy a 20-year bond to pick up a short-term profit as to buy a 3-month security. Strict proponents of this view argue that the shape of the Treasury yield curve is therefore determined only by market expectations about future interest rates. This position has been called the *pure expectations theory* of the term structure of interest rates.

The **pure expectations theory** (often simply referred to as the “expectations theory”) assumes that: (1) bond traders establish bond prices and interest rates strictly on the basis of expectations for future interest rates, and (2) they are indifferent to maturity because they do not view long-term bonds as being riskier than short-term bonds. If this were true, then the maturity risk premium (MRP) would be zero, and long-term interest rates would simply be a weighted average of current and expected future short-term interest rates.

To illustrate this point, assume that 1-year Treasury securities currently yield 5% while 2-year Treasury securities yield 5.5%. Investors with a 2-year horizon have two primary options, as follows:

Option 1: Buy a 2-year security and hold it for 2 years.

Option 2: Buy a 1-year security, hold it for 1 year, and then, at the end of the year, reinvest the proceeds in another 1-year security.

If investors select Option 1, then for every dollar they invest today they will have accumulated \$1.113025 by the end of Year 2:

$$\text{Funds at end of Year 2} = \$1(1.055)^2 = \$1.113025$$

Investors who select Option 2 must expect to end up with at least the same amount (else they would be indifferent between the options):

$$\text{Funds at end of Year 2} = \$1(1.05)(1 + X)$$

Here X is the expected interest rate on a 1-year Treasury security one year from now.

If the expectations theory is correct, then each option must provide the same amount of cash at the end of 2 years, which implies that:

$$(1.05)(1 + X) = (1.055)^2$$

We can rearrange this equation and then solve for X :

$$1 + X = (1.055)^2/1.05$$

$$X = (1.055)^2/1.05 - 1 = 0.0600238 = 6.00238\%$$

Thus, if the expectations theory is correct, the current yield curve would indicate that the market expects the 1-year rate to be 6.00238% 1 year from now.

We can use yield curve data to help predict future short-term interest rates. In the absence of maturity risk premiums, an upward-sloping Treasury yield curve implies that the market expects future short-term rates to increase. Also, notice that the 2-year rate of 5.5% is a geometric average of the current and expected 1-year rates.¹ To understand the logic behind the averaging process, ask yourself what would happen if long-term yields were *not* an average of expected short-term yields. For example, suppose investors expected the 1-year Treasury rate to be 6.00238% a year from now, yet 2-year bonds yielded only 5.25%. Bond traders would be able to earn a profit by adopting the following strategy:

1. Borrow money for 2 years at an annual cost of 5.25%.
2. Invest the money in a series of 1-year securities. The annual expected return over the 2-year period would be $[(1.05)(1.0600238)]^{1/2} - 1 = 5.5\%$.

Earning 5.5% when the cost is 5.25% is a good deal, so bond traders would rush to borrow money (demand funds) in the 2-year market and invest (or supply funds) in the 1-year market. Recall that an increase in the demand for funds raises interest rates whereas an increase in the supply of funds reduces interest rates. Therefore, bond traders would push up the 2-year yield but reduce the yield on 1-year bonds. The net effect would be a market equilibrium in which the 2-year rate is a weighted average of expected future 1-year rates.

The preceding analysis is based on the assumption that the maturity risk premium is zero, but most evidence suggests that there is a positive maturity risk premium. For example, assume once again that 1- and 2-year maturities yield 5.0% and 5.5%, respectively, so we have a rising yield curve. However, now assume that the maturity risk premium on the 2-year bond is 0.2% versus 0% for the 1-year bond. This premium means that in equilibrium the expected annual return on a 2-year bond (5.5%) must be 0.2% higher than the expected return on a series of two 1-year bonds (5% and X), so the expected return on the series must be $5.5\% - 0.2\% = 5.3\%$:

1. The geometric average of the current and expected 1-year rates can be expressed as $[(1.05)(1.0600238)]^{1/2} - 1 = 0.055$ or 5.50%. The arithmetic average of the two rates is $(5\% + 6.00238\%)/2 = 5.50119\%$. The geometric average is theoretically superior, but the difference is only 0.00119%. With interest rates at the levels they have been in the United States and most other nations in recent years, the geometric and arithmetic averages are so close that many people simply use the arithmetic average, especially given the other assumptions that underlie the estimation of future 1-year rates.

$$\begin{aligned}
 \text{Expected return on 2-year series} &= \text{Rate on 2-year bond} - \text{MRP} \\
 &= 0.055 - 0.002 \\
 &= 0.053 = 5.3\%
 \end{aligned}$$

Now recall that the annual expected return from the series of two 1-year securities can be expressed as follows, where X is the 1-year rate next year:

$$\begin{aligned}
 (1.05)(1 + X) &= (1 + \text{Expected return on 2-year series})^2 = (1.053)^2 \\
 1.05X &= (1.053)^2 - 1.05 \\
 X &= \frac{0.0588090}{1.05} \\
 &= 0.0560086 = 5.60086\%
 \end{aligned}$$

Therefore, in this scenario market participants must expect the 1-year rate next year to be 5.60086%.

Observe that the yield curve rises by 0.5% when the years to maturity increases from 1 to 2: $5.5\% - 5.0\% = 0.5\%$. Of this 0.5% increase, 0.2% is attributable to the MRP and the remaining 0.3% is due to the expected increase in 1-year rates next year.

Putting all this together, we see that we can use the yield curve to predict the short-term rates that will exist for next year, but to do this we must have an estimate of the maturity risk premium. If our estimate of the MRP is incorrect, then so will be our yield curve-based interest rate forecast. Thus, although the yield curve can be used to obtain insights into the direction of future interest rates, we cannot back our expected interest rates with precision unless either the pure expectations theory holds exactly or we know with certainty the exact maturity risk premium. Because neither of these conditions holds, forecasts of future interest rates are only approximations—though they may be good approximations.

What key assumption underlies the pure expectations theory?

Assuming that the pure expectations theory is correct, how are expected short-term rates used to calculate expected long-term rates?

According to the pure expectations theory, what would happen if long-term rates were *not* an average of expected short-term rates?

Most evidence suggests that a positive maturity risk premium exists. How does this affect your calculations when determining interest rates?

Assume the interest rate on a 1-year T-bond is currently 7% and the rate on a 2-year bond is 9%. If the maturity risk premium is 0%, what is a reasonable forecast of the rate on a 1-year bond next year? **(11.04%)** What would the forecast be if the maturity risk premium on the 2-year bond were 0.5% and the maturity premium were 0% for the 1-year bond? **(10.02%)**

SELF TEST

Problems

4D-1 Expectations Theory

One-year Treasury securities yield 5%. The market anticipates that 1 year from now, 1-year Treasury securities will yield 6%. If the pure expectations theory is correct, what is the yield today for 2-year Treasury securities?

4D-2 Expectations Theory

Interest rates on 4-year Treasury securities are currently 7%, while 5-year Treasury securities yield 7.5%. If the pure expectations theory is correct, what does the market believe that 2-year securities will be yielding 4 years from now?

4D-3 Expectations Theory and Inflation

Suppose 2-year Treasury bonds yield 4.5% while 1-year bonds yield 3%. The maturity risk premium is 0% and r^* is 1%.

- Using expectations theory, what is the yield on a 1-year bond 1 year from now?
- What is the expected inflation rate in Year 1? Year 2?

4D-4 Expectations Theory

Assume that the real risk-free rate is 2% and that the maturity risk premium is 0%. If the 1-year bond yield is 5% and a 2-year bond (of similar risk) yields 7%, then what is the 1-year interest rate expected for Year 2? What inflation rate is expected during Year 2? Comment on why the average interest rate during the 2-year period differs from the 1-year interest rate expected for Year 2.